actions. Whereas the Peaslee[18] and True and Ford[20] interactions do not have any triplet component, the interactions determined by TWP,[21] Raz and French[22] and BKS[23] vary from 0.2 to 0.4. It must however be remembered that these interactions have been derived from various available data such as nuclear energy levels, transition probabilities, magnetic moments and stripping reactions. Though it is obvious that any of these properties must be satisfactorily explained by a given set of parameters, due to the approximations that are involved in determining these properties (and the insufficient knowledge about the nucleon-nucleon potential), the situation becomes complicated. In other words, forms of interactions are different as one goes from one property of the nucleus to another, which is not at all surprising. It should also be mentioned that while the parameters of Barker[12] and Peaslee[18] have been determined from the analysis of $p_{1/2}$ and $s_{1/2}$ doublets in the $A \approx 16$ region which would not involve any configuration mixing as far as the $\frac{1}{2} \times j$ doublets are concerned, the effect of admixtures has been quite predominant as far as the quantitative agreement of the positions of the energy levels are concerned. The analysis

of our work in Secs. III and IV differs from the rest of the authors in two ways: (1) while the configuration mixing is entirely neglected by Dawson, Talmi, and Walecka,[9] the triplet forces are entirely neglected by Peaslee[18] and True and Ford.[20] (2) The nature of the interaction is assumed the same in all the configurations. It has however been indicated by Thankappan, Waghmare, and Pandya[21] that the two-body effective interaction in Zr^{90} is configuration-dependent. This is more evident from our present analysis where we take into account both the singlet as well as triplet forces and the effect of configuration mixing as well. In view of the calculations on the many-body systems, such an effect may not be observed in Be^{10}. However, it is certainly important in the case of Ni^{58} where the first excited state in Ni^{57} lies close to the ground state. It is thus clear that the interactions that we have derived in subsequent sections determine the nature of the effective nucleon-nucleon potential. It is also clear that it is not possible, at this stage, to get such an information about the d–s shells.

ACKNOWLEDGMENTS

[20] W. W. True and K. W. Ford, Phys. Rev. 109, 1675 (1958).
[21] V. K. Thankappan, Y. R. Waghmare, and S. P. Pandya, Progr. Theoret. Phys. (Kyoto) 26, 22 (1961).
[22] B. J. Raz and J. B. French, Phys. Rev 104, 1411 (1956).
[23] I. M. Band, Yu I. Kharitonov, and L. A. Sliv, Nucl. Phys. 35, 136 (1962).

Velocity-Dependent Potentials and the Shell Model of Oxygen-18

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An expansion of the shell-model matrix elements of the velocity-dependent potential $m^{-1}[p^2 V(r) + V(r) p^2]$ in the Talmi integrals of V is derived and applied to calculate the energy levels of O^{18} using the nucleon-nucleon potential of Green. It is found that the correct ordering of the levels is obtained but the potential must be altered slightly to obtain agreement comparable with that given by Dawson, Talmi, and Walecka using the Brueckner-Gammel-Thaler potential.

1. INTRODUCTION

THE possibility that velocity-dependent potentials (v.d.p.) could replace the hard core of the nucleon-nucleon potential, permitting more tractable calculations in many-body problems, was suggested by Peierls[1] at the Kingston Conference. It has since been discussed by many authors.[2]

Green's calculations are the most extensive, and they have been supplemented by Preston, Armstrong, and Bhaduri. The phase-shift data were fitted quite well, although the agreement obtained is probably not the best possible. The triplet odd parameters, in particular, could be readjusted with advantage. The potential used by these authors was of the form

$$-V(r) + m^{-1}(p^2 \omega(r) + \omega(r) p^2),$$

[1] R. E. Peierls, *Proceedings of the International Conference on Nuclear Structure, Kingston, 1960*, edited by D. A. Bromley and E. W. Vogt (North-Holland Publishing Company, Amsterdam, 1960), p. 7.
[2] M. Razavy, G. Field, and J. S. Levinger, Phys. Rev. 125, 269 (1962); O. Rojo and L. M. Simmons, *ibid.* 125, 273 (1962); A. M.

Green, Nucl. Phys. 33, 218 (1962); M. A. Preston, P. J. Armstrong, and R. K. Bhaduri, Phys. Letters 2, 183 (1962); E. Werner, Nucl. Phys. 35, 324 (1962); F. Peischl and E. Werner, *ibid.* 43, 372 (1963).

where V is a mixture of Gaussian and Yukawa forms, and includes tensor and spin-orbit forces, and ω is a Gaussian potential. In view of the success obtained in representing the two-body data with such a potential, it would seem to be useful to do shell-model calculations using it.

O^{18} has been investigated in the shell model by various authors. The early work of Elliott and Flowers[3] showed that reasonable agreement with the experimental level structure could be obtained using a Rosenfeld type potential which fitted the low-energy two-body data, and that the ground state was at too high an energy, unless the $1d_{3/2}$ configuration were included. Similar results were obtained by Dawson, Talmi, and Walecka,[4] who used the Brueckner-Gammel-Thaler potential, treating the hard cores by the Bethe-Goldstone method. There have also been some attempts to obtain the matrix elements of the residual interaction from the energy levels of O^{18}, O^{19}, and O^{20}. The most recent attempt is that of Pandya.[5] He included only $1d_{5/2}$ and $2s_{1/2}$ configurations, so the results of Elliott and Flowers and D.T.W. would suggest that he would overestimate in magnitude the matrix element

$$\langle (1d_{5/2})^2 J=0 \mid V \mid (1d_{5/2})^2 J=0 \rangle.$$

In all cases, the interaction of the two outer neutrons with the core is taken empirically from the position of the $\frac{5}{2}^+$, $\frac{1}{2}^+$, and $\frac{3}{2}^+$ levels in the O^{17} spectrum, shown in Fig. 1. The core energy and the contribution of the particle-core interaction is subtracted out of the experimental data for comparison with the results of the calculation. In Fig. 1 the ground state of O^{17} is placed at an energy

$$B.E.(O^{17}) - B.E.(O^{16})$$

and the ground state of O^{18} at the energy

$$B.E.(O^{18}) - B.E.(O^{16}) - 2\{B.E.(O^{17}) - B.E.(O^{16})\}.$$

A shell-model calculation of the energy levels of O^{18} using the velocity-dependent potential of Green has been carried out. It is found that this potential leads to matrix elements which are similar to and slightly less than those obtained by D.T.W. The energy levels have been calculated using $1d_{5/2}$ and $2s_{1/2}$ configurations only, diagonalizing the matrices in Table I, and are found to be correctly ordered but to lie rather high.

A detailed comparison with the results of D.T.W. suggests that an increase in the strength of the Gaussian singlet potential and a decrease in the strength of the velocity-dependent part would be the simplest way to improve the agreement. It is not at present known whether it is possible to choose a potential which would improve the agreement with the energy levels and not
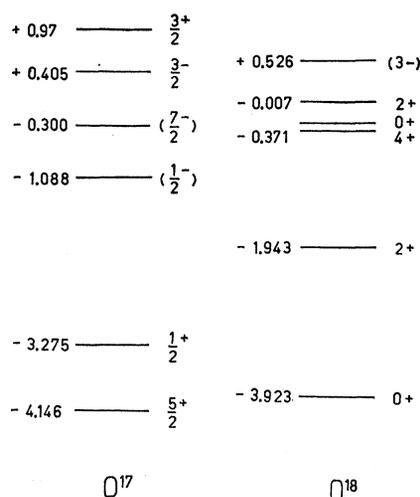
[3] J. P. Elliott and B. H. Flowers, Proc. Roy. Soc. (London) **229**, 536 (1955).
[4] J. F. Dawson, I. Talmi, and J. D. Walecka, Ann. Phys. (N. Y.) **18**, 339 (1962). Referred to as D.T.W.
[5] S. P. Pandya, Nucl. Phys. **43**, 636 (1963).

FIG. 1. Observed energy levels of O^{17} and O^{18} (energies in MeV).

be inconsistent with the two-body phase shifts. However, it appears that the answer to this problem will be more straightforward with velocity-dependent potentials than with hard-core potentials.

2. CALCULATION OF MATRIX ELEMENTS

Because the isotopic spin of O^{18} is unity, only the singlet even and triplet odd potentials contribute. These potentials will be denoted by V_0 and V_1. Green's potential has the form

$$V_0 = V_0{}^c(r) + m^{-1}(p^2 V_0{}^P(r) + V_0{}^P(r)p^2),$$

$$V_1 = V_1{}^c(r) + V_1{}^v(r)\mathbf{l}\cdot\mathbf{s} + V_1{}^t(r)S_{12},$$

where $\mathbf{r}$ and $\mathbf{p}$ are the relative position and momentum of the two nucleons, and $\mathbf{l}\cdot\mathbf{s}$ and S_{12} are the usual spin-orbit and tensor operators.[6]

TABLE I. Hamiltonian matrices for O^{18}, including the $1d_{5/2}$ and $2s_{1/2}$ configurations.

$$E_J = \langle (1d_{5/2})^2 J \mid V \mid (1d_{5/2})^2 J \rangle$$
$$E_0' = \langle (2s_{1/2})^2 J=0 \mid V \mid (2s_{1/2})^2 J=0 \rangle$$
$$E_J' = \langle 1d_{5/2}2s_{1/2}J \mid V \mid 1d_{5/2}2s_{1/2}J \rangle \quad \text{for} \quad J=2, 3$$
$$F_0 = \langle (1d_{5/2})^2 J=0 \mid V \mid (2s_{1/2})^2 J=0 \rangle$$
$$F_2 = \langle (1d_{5/2})^2 J=2 \mid V \mid 1d_{5/2}2s_{1/2}J=2 \rangle$$

Δ is the difference between the $\frac{5}{2}^+$ and $\frac{1}{2}^+$ levels in the O^{17} spectrum.

1.	$J=0$	E_0	F_0
		F_0	$F_0'+2\Delta$
2.	$J=2$	E_2	F_2
		F_2	$E_2'+\Delta$
3.	$J=3$	$E_3'+\Delta$	
4.	$J=4$	E_4	

[6] M. A. Preston, *Physics of the Nucleus* (Addison-Wesley Publishing Company, Inc., Reading, Massachusetts, 1962), Chap. 5.

TABLE II. Parameters of Green's potential. The radial potential is
$V(r) = -A \exp(-r^2/c^2) - B(\mu r)^{-1}e^{-\mu r}(1-e^{-\alpha\mu r})$.

Force	A	c	B	α
Singlet central	49.1 MeV	1.268 F	11.02 MeV	6
Triplet central			−3.69 MeV	6
Triplet spin orbit	82.9 MeV	1.04 F		
Triplet tensor			−14.5 MeV	6
Singlet v.d.p.	−1.14	0.695 F		

For the radial factors $V(r)$, Green assumed a form

$$V(r) = -A \exp(-r^2/c^2) - B(\mu r)^{-1}e^{-\mu r}(1-e^{-\alpha\mu r}),$$

with $\mu = 0.7082$ F^{-1}. The parameters of his singlet even potential (a) and triplet odd potential (a), used in the present calculation, are given in Table II.

The matrix elements of the potential have been obtained using the transformation to relative motion and center-of-mass states first discussed by Talmi.[7] The notation of Brody and Moshinsky[8] has been followed. The relative and center-of-mass coordinates they introduce are slightly unusual, being defined by

$$\mathbf{r}' = 2^{-1/2}(\mathbf{r}_1-\mathbf{r}_2), \quad \mathbf{p}' = 2^{-1/2}(\mathbf{p}_1-\mathbf{p}_2),$$
$$\mathbf{R}' = 2^{-1/2}(\mathbf{r}_1+\mathbf{r}_2), \quad \mathbf{P}' = 2^{-1/2}(\mathbf{p}_1+\mathbf{p}_2).$$

The transformation from $\mathbf{r}_1$ and $\mathbf{r}_2$ to $\mathbf{r}'$ and $\mathbf{R}'$ leaves the harmonic oscillator Hamiltonian invariant in form. $|n_1 l_1 n_2 l_2 LM\rangle$ is the unsymmetrized two-particle state, with particle 1 in the state $n_1 l_1$ and particle 2 in the state $n_2 l_2$, the particles being coupled to a total angular momentum L, M. $|nkNKLM\rangle$ is the state with the relative motion characterized by the quantum numbers nk and the center-of-mass motion by NK, the total angular momentum being L, M. We may transform from one representation to the other

$$|n_1 l_1 n_2 l_2 LM\rangle = \sum |nkNKLM\rangle(nkNKL|n_1 l_1 n_2 l_2 L),$$

where the summation is over all values of n, k, N, K, which are consistent with the conservation laws. The transformation brackets $(nkNKL|n_1 l_1 n_2 l_2 L)$, which are known to be independent[9] of M are those tabulated by Brody and Moshinsky.[8] They also tabulate the coefficients $B(n'k'; nk; p)$ of the expansion in terms of the Talmi integrals $I_p[V]$ of $(n'k'\|V\|nk)$,[10] the reduced matrix elements between relative motion states of a

central potential $V(r)$.

$$(n'k'\|V\|nk) = \sum_p B(n'k'; nk; p)I_p[V],$$

$$I_p[V] = \frac{2}{(p+\tfrac{1}{2})!}\int_0^\infty e^{-x^2}x^{2(p+1)}V(\sqrt{2}ax)dx,$$

where $a = (\hbar/m\omega)^{1/2}$ is the harmonic oscillator length parameter. The value $a = 1.71$ F [11] has been used in this calculation. Analytic expressions for $I_p[V]$ are given by Thieberger[12] for various common potentials. Those required are

$$I_p[\exp(-r^2/c^2)] = (1+2a^2/c^2)^{-(p+\frac{3}{2})},$$
$$I_p[e^{-\mu r}/r] = 2^{p+\frac{1}{2}}\pi^{-1/2}a^{-1}(p!)e^{\frac{1}{2}a^2\mu^2}Hh_{2p+1}(a\mu),$$

where $Hh_n(z)$ is the Hermitian probability integral defined by[13]

$$n!Hh_n(z) = \int_0^\infty \exp[-\tfrac{1}{2}(z+t)^2]t^n dt.$$

The properly symmetrized states $|\ \rangle$ are built from the unsymmetrized states $|\ )$ in the usual way. With β as a shorthand for n, l, j

$$|\beta_1\beta_2 JMT\rangle = 2^{1/2}[|\beta_1\beta_2 JM) + (-1)^{j_1+j_2-J+T}|\beta_2\beta_1 JM)]$$

if $\beta_1 \neq \beta_2$ and

$$|\beta^2 JMT\rangle = \tfrac{1}{2}[1+(-1)^{2j-J+T}]|\beta^2 JM).$$

j-j coupled matrix elements are in turn obtained from L-S coupled matrix elements, which are calculated using Racah algebra and the Talmi transformations. α is used to represent the quantum numbers $n_1 l_1 n_2 l_2$ and $\mathcal{U}_\sigma{}^\lambda$ for the σ, λ part of the potential, for example

$$\mathcal{U}_0{}^P = m^{-1}(p^2 V_0{}^P(r) + V_0{}^P(r)p^2)$$

and

$$\mathcal{U}_1{}^v = V_1{}^v(r)\mathbf{l}\cdot\mathbf{s}.$$

The L-S matrix elements of $\mathcal{U}_\sigma{}^\lambda$ are expanded as a series of Talmi integrals

$$(\alpha'L'S'JM|\mathcal{U}_\sigma{}^\lambda|\alpha LSJM)$$
$$= \delta_{S'S}\delta_{S\sigma}\sum_p C_\sigma{}^\lambda(\alpha'L'; \alpha L; J:p)I_p[V_\sigma{}^\lambda].$$

The results for central spin-orbit and tensor forces are given by Brody and Moshinsky.

$$C_\sigma{}^c(\alpha'L; \alpha L; J:p)$$
$$= \delta_{L'L}\sum_{nn'k} B(n'k; nk; p)$$
$$\times \sum_{NK} (n'kNKL|\alpha'L)(nkNKL|\alpha L),$$

[7] I. Talmi, Helv. Phys. Acta **25**, 185 (1952).

[8] T. A. Brody and M. Moshinsky, *Tables of Transformation Brackets* (Monografias del Institio de Fisica, Mexico, 1960).

[9] E. U. Condon and G. H. Shortley, *The Theory of Atomic Spectra* (Cambridge University Press, Cambridge, England, 1963), p. 49.

[10] The reduced matrix element is defined here so that the matrix elements and reduced matrix elements of a scalar are identical. See Brink and Satchler, *Angular Momentum* (Clarendon Press, Oxford, England, 1962), p. 57.

[11] P. Goldhammer, Rev. Mod. Phys. **35**, 40 (1963).

[12] R. Thieberger, Nucl. Phys. **2**, 533 (1957).

[13] *British Association Mathematical Tables* (Cambridge University Press, Cambridge, England, 1946), 2nd ed., Vol. 1, H. Jefferies and B. S. Jefferies, *Methods of Mathematical Physics* (Cambridge University Press, Cambridge, England, 1956), 3rd ed., p. 622.

$C_1^v(\alpha'L;\alpha L;J:p)$

$= (-1)^J \begin{Bmatrix} L' & 1 & J \\ 1 & L & 1 \end{Bmatrix} [6(2L'+1)(2L+1)]^{1/2}$

$\times \sum_{nn'k} B(n'k;nk;p)A(\alpha'L';\alpha L;n'nk)$,

where

$A(\alpha'L';\alpha L;n'nk)[k(k+1)(2k+1)]^{-1/2}$

$= \sum_{NK} (n'kNKL'|\alpha'L')(nkNKL|\alpha L)$

$\times (-1)^{k+K} \begin{Bmatrix} k & L' & K \\ L & k & 1 \end{Bmatrix}$,

and

$C_1^t(\alpha'L';\alpha L;J:p)[120(2L'+1)(2L+1)]^{-1/2}$

$= (-1)^{1+J} \begin{Bmatrix} L' & 1 & J \\ 1 & L & 2 \end{Bmatrix} \sum_{n'k'nk} B(n'k';nk;p)$

$\times D(\alpha'L';\alpha L;n'k';nk)$,

where

$D(\alpha'L';\alpha L;n'k';nk)[(2k'+1)(2k+1)]^{-1/2}$

$= \sum_{NK} (n'k'NKL'|\alpha'L')(nkNKL|\alpha L)$

$\times (-1)^{k+k'+K} \begin{pmatrix} k' & k & 2 \\ 0 & 0 & 0 \end{pmatrix} \begin{Bmatrix} k' & L' & K \\ L & k & 2 \end{Bmatrix}$.

The $3j$ and $6j$ symbols are defined with the same phase as in Brink and Satchler.[10]

The calculation of the C coefficients for the velocity-dependent potential is outlined in the next section.

3. THE VELOCITY-DEPENDENT POTENTIAL

The velocity-dependent term in the potential is, when written in terms of $\mathbf{r}'$ and $\mathbf{p}'$ (omitting the sub- and superscripts),

$$\mathcal{v} = \tfrac{1}{2}m^{-1}(p'^2 V(\sqrt{2}r') + V(\sqrt{2}r')p'^2).$$

Using the Talmi transformation we obtain

$(\alpha'LSJM|\mathcal{v}|\alpha LSJM) = \sum_{n'nk} (n'k||\mathcal{v}||nk)$

$\times \sum_{NK} (n'kNKL|\alpha'L)(nkNKL|\alpha L)$.

It is obvious that

$(n'k||p'^2V||nk) = \sum_{n''k''} (n'k||p'^2||n''k'')(n''k''||V||nk)$

and

$(n'k||Vp'^2||nk) = \sum_{n''k''} (n'k||V||n''k'')(n''k''||p'^2||nk)$,

so the essential step in finding the matrix elements of $\mathcal{v}$ is the determination of the matrix elements of p'^2.

There are many ways to do this. Possibly the simplest makes use of the relation

$(n'k||\tfrac{1}{2}m^{-1}p'^2 + \tfrac{1}{2}m\omega^2 r'^2||nk) = (2n+k+\tfrac{3}{2})\hbar\omega\delta_{n'n}$

to relate the matrix elements of p'^2 to those of r'^2. The wave function $(r'\theta\phi|nkm)$ is given by

$$(r'\theta\phi|nkm) = R_{nk}(r'/a)Y_{km}(\theta,\phi),$$

where $Y_{km}(\theta,\phi)$ is the normalized spherical harmonic with the Condon and Shortley definition of phases,

$R_{nk}(x) = \{2(k+\tfrac{3}{2})_n/n!(k+\tfrac{1}{2})!\}^{1/2}x^k e^{-\frac{1}{2}x^2}a^{-3/2}$
$\times {}_1F_1(-n;k+\tfrac{3}{2};x^2)$,

$(a)_n = a(a+1)\cdots(a+n-1)$,

and ${}_1F_1(a;b;z)$ is the Kummer confluent hypergeometric function. Using the recurrence relation[14]

$a\,{}_1F_1(a+1;b;z) = (z+2a-b)\,{}_1F_1(a;b;z)$
$+ (b-a)\,{}_1F_1(a-1;b;z)$,

we can show that

$[x^2 - (2n+k+\tfrac{3}{2})]R_{nk}(x) = -(n+1)^{1/2}(n+k+\tfrac{3}{2})^{1/2}$
$\times R_{n+1,k}(x) - n^{1/2}(n+k+\tfrac{1}{2})^{1/2}R_{n-1,k}(x)$.

The reduced matrix elements of r'^2 are therefore given by

$a^{-2}(n'k||r'^2||nk) - (2n+k+\tfrac{3}{2})\delta_{n'n}$
$= -n^{1/2}(n+k+\tfrac{1}{2})^{1/2}\delta_{n',n-1}$
$- (n+1)^{1/2}(n+k+\tfrac{3}{2})^{1/2}\delta_{n',n+1}$,

giving

$[a^2/\hbar^2](n'k||p'^2||nk) - (2n+k+\tfrac{3}{2})\delta_{n'n}$
$= n^{1/2}(n+k+\tfrac{1}{2})^{1/2}\delta_{n',n-1}$
$+ (n+1)^{1/2}(n+k+\tfrac{3}{2})^{1/2}\delta_{n',n+1}$.

The coefficient $C_\sigma^P(\alpha'L';\alpha L;J:p)$ may then be expressed in the form

$C_\sigma^P(\alpha'L';\alpha L;J:p) = \dfrac{\hbar^2}{2ma^2}\delta_{L'L}\sum_{n'nkNK} (n'kNKL|\alpha'L)$

$\times (nkNKL|\alpha L)\Gamma(n'nk:p)$,

where

$\Gamma(n'nk;p) = \sum_{st} B(sk;tk;p)\gamma(n'nk;st)$,

and the γ coefficient is given algebraically in Table III.

TABLE III. The coefficient $\gamma(n'nk;st)$.

s \ t	$n-1$	n	$n+1$
$n'-1$		$n'^{1/2}(n'+k+\tfrac{1}{2})^{1/2}$	
n'	$n^{1/2}(n+k+\tfrac{1}{2})^{1/2}$	$2(n'+n)+2k+3$	$(n+1)^{1/2}(n+k+\tfrac{3}{2})^{1/2}$
$n'+1$		$(n'+1)^{1/2}(n'+k+\tfrac{3}{2})^{1/2}$	

[14] I. N. Sneddon, *Special Functions of Mathematical Physics and Chemistry* (Oliver and Boyd, Edinburgh, Scotland, 1961), 2nd ed., p. 38.

The velocity-dependent potential is very short ranged, so the Talmi integrals decrease rapidly as p increases, and therefore $C^P(\ :p)$ is required for small values of p only. In $\Gamma(n'nk\,;p)$ p takes the values k, $k+1$, $\cdots$, $k+n'+n$. Algebraic formulas for the B coefficients are used to derive an algebraic expression for $\Gamma(n'nk\,;k+q)$ which is simple for small q. The general expression given by Brody, Jacob, and Moshinsky[15] for B is very complicated, but more straightforward formulas may be derived for

$$B(n'k\,;nk\,;k+q)$$

for small q. When $q=0$

$$B(n'k\,;nk\,;k)=\{(k+\tfrac{3}{2})_{n'}(k+\tfrac{3}{2})_n/n'\,!n\,!\}^{1/2}\,,$$

and for $q\neq0$ define

$$B(n'k\,;nk\,;k+q)=b(n'nk\,;q)B(n'k\,;nk\,;k)\,.$$

Then

$$-b(n'nk\,;1)=(n'+n)\,,$$

$$b(n'nk\,;2)=\frac{n'(n'-1)+n(n-1)}{2!}+n'n\frac{2k+5}{2k+3}\,,$$

$$-b(n'nk\,;3)=\frac{n'(n'-1)(n'-2)+n(n-1)(n-2)}{3!}$$
$$+\frac{n'n(n'+n-2)}{2!}\frac{2k+7}{2k+3}\,.$$

Substituting in the equation for Γ, the following expressions are obtained:

$$\Gamma(n'nk\,;p)=B(n'k\,;nk\,;k)\Gamma'(n'nk\,;p)\,,$$

$$\Gamma'(n'nk\,;k)=4(n'+n)+4k+6\,,$$

$$-\Gamma'(n'nk\,;k+1)=[4(n'+n)+4k+6](n'+n)+2k+3\,,$$

$$\Gamma'(n'nk\,;k+2)=-2n'n-(n'+n)(n'+n+k+\tfrac{1}{2})$$
$$+2(n'^2+n^2)(n'+n+k+1)$$
$$+[4(n'+n)+4k+6]n'n\frac{2k+5}{2k+3}$$
$$+\tfrac{1}{2}(n'+n)(2k+5)\,.$$

In this calculation we retained only the terms in $I_0[V^P]$ and $I_1[V^P]$. This is equivalent to the assumption that the velocity-dependent potential is sufficiently short ranged to affect the s and p states only, and is to be compared with the approximation of D.T.W. that the hard core affects only the s states.

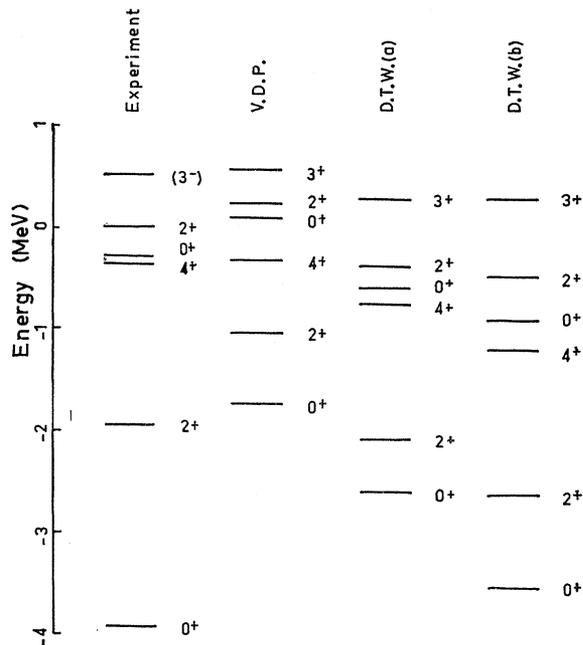15 T. A. Brody, G. Jacob, and M. Moshinsky, Nucl. Phys. **17**, 16 (1960).

FIG. 2. Energy levels of O^{18}. v.d.p. given by the present calculation using Green's potential and including $1d_{5/2}$ and $2s_{1/2}$ configurations. D.T.W. (a): Calculated by D.T.W. using $1d_{5/2}$ and $2s_{1/2}$ configurations. D.T.W. (b): Calculated by D.T.W. using $1d_{5/2}$, $2s_{1/2}$, and $1d_{3/2}$ configurations.

4. RESULTS

If the series to first order for

$$(n'k\|\mathcal{U}^c\|nk)$$

and

$$(n'k\|\mathcal{U}^P\|nk)$$

are compared,

$$(n'k\|\mathcal{U}^c\|nk)=B(n'k\,;nk\,;k)I_k[V^c]+\cdots$$
$$(n'k\|\mathcal{U}^P\|nk)=[4(n'+n)+4k+6]B(n'k\,;nk\,;k)$$
$$\times I_k[V^P]+\cdots\,,$$

it is apparent that the velocity-dependent potential cannot be taken into account by a modification of $I_k[V^c]$ because the contribution is dependent on n and n'. D.T.W. took the hard core into account by modifying $I_0[V^c]$. In principle, this gives a means of

TABLE IV. Matrix elements of the residual interaction.

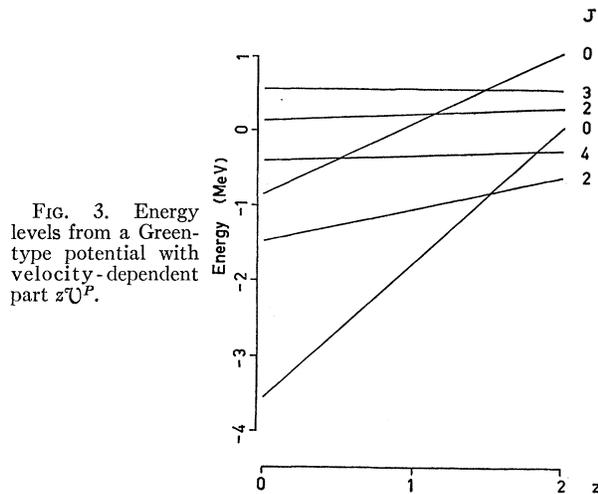| Matrix element	Present calculation	D.T.W.	Pandya
E_0	−1.558	−1.964	−3.00
E_2	−0.715	−1.602	−1.20
E_4	−0.337	−0.766	−0.37
E_0'	−1.834	−3.019	−2.90
E_2'	−0.986	−1.689	−1.65
E_3'	−0.312	−0.649	+1.50
F_0	−0.515	−0.952	−1.50
F_2	−0.560	−0.780	−0.93

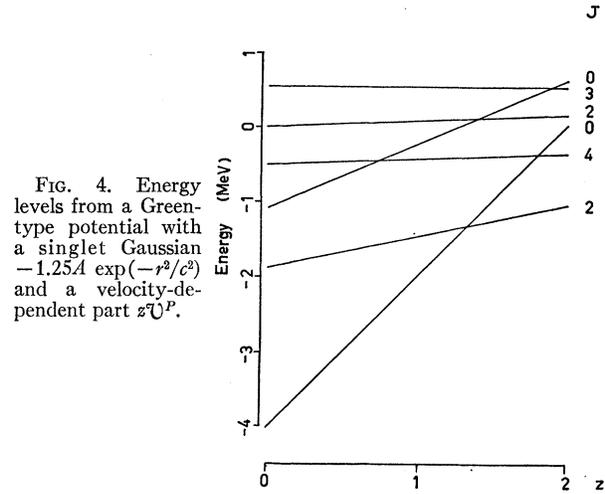FIG. 3. Energy levels from a Green-type potential with velocity-dependent part $z\mathcal{U}^P$.

FIG. 4. Energy levels from a Green-type potential with a singlet Gaussian $-1.25A \exp(-r^2/c^2)$ and a velocity-dependent part $z\mathcal{U}^P$.

distinguishing the two potentials. However, comparing the energy levels D.T.W. (a) obtained by D.T.W. using the $1d_{5/2}$ and $2s_{1/2}$ configurations, and the levels v.d.p. given by Green's potential in Fig. 2, it is difficult to discern the difference. The necessity of including the $1d_{3/2}$ states is illustrated by the comparison of the levels D.T.W. (a), which omit them, and D.T.W. (b), which include them.

To illustrate the effect of changing the potential, Fig. 3 shows the energy levels for a potential similar to Green's, but with the velocity-dependent part multiplied by a factor z plotted as a function of z. Figure 4 is similar, but in this case the Gaussian part of the static singlet potential has been multiplied by a factor 1.25. This suggests that a better fit can be obtained by

TABLE V. Contributions to matrix elements $\langle (1d_{5/2})^2 J | V | (1d_{5/2})^2 J \rangle$.

J		Singlet central	Central	Triplet spin-orbit	Tensor	Total
0	v.d.p.	−2.835	+0.093	+0.452	+0.732	−1.558
	D.T.W.	−3.209	−0.145	+0.557	+0.833	−1.964
2	v.d.p.	−0.770	+0.114	−0.170	+0.111	−0.714
	D.T.W.	−1.183	−0.176	−0.300	+0.057	−1.602
4	v.d.p.	−0.481	+0.112	−0.102	+0.133	−0.338
	D.T.W.	−0.671	−0.150	−0.098	+0.153	−0.766

increasing the static singlet strength and decreasing the velocity-dependent strength.

This same conclusion may be arrived at by a more detailed comparison with the results of D.T.W. The matrix elements derived from Green's potential, those of D.T.W. and of Pandya are given in Table IV. The present matrix elements are consistently less in magnitude than those of D.T.W. (Table V) which shows the contribution of the various parts of the potential to the matrix elements $\langle (1d_{5/2})^2 J | V | (1d_{5/2})^2 J \rangle$ suggests that the singlet potential is too weak.

While variation of the singlet potential seems to be necessary to obtain results comparable with those of D.T.W., it is possible that variation of the triplet potential would also improve the fit to the observed energy levels. It is, at present, an open question whether it is possible to alter the potential, at the same time to improve the agreement both with the energy levels of nuclei in the shell model and the two-body data.

ACKNOWLEDGMENTS